Simulation of the Wave of Action Potential Propagation over the Length of the Axon at Various Times by Comsol Multiphysics Software

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Abstract – In this study, simulation of Fitzhugh nagumo model in order to generation and propagation of the wave of action potential over the length the of the squid giant axon and survey the passage of time effect in wave propagation have been explored. Simulation has been performed by using the Comsol Multiphysics 3.5 a software. So that, in this simulation, squid giant axon is intended to form hollow cylinder. For this purpose PDE (general form) module have been used. Then by running the simulation steps, we see that the wave of action potential along axon at different times released and from the beginning to the end of the axon is transmitted. Thus, nerve signals are transmitted between different organs.

Key Words: Action Potential; Axon; Nerve Signal; Fitzhugh Nagumo Mode; Comsol Multiphysics.

1 Introduction

The nervous system coordinates the activities of many other organ systems. It activates muscles for movement, controls the secretion of hormones from glands, regulates the rate and depth of breathing, and is involved in modulating and regulating a multitude of other physiological processes. To perform these functions, the nervous sys system relies on neurons, which are designed for the rapid transmission of information from one cell to another by conducting electrical impulses and secreting chemical neurotransmitters. The electrical impulses propagate along the length of nerve fiber processes to their terminals, where they initiate a series of events that cause the release of chemical neurotransmitters. The release of neurotransmitters occurs at sites of synaptic contact between two nerve cells. Released neurotransmitters bind with their receptors on the postsynaptic cell membrane. The activation of these receptors either excites or inhibits the postsynaptic neuron. The propagation of action potentials, the release of neurotransmitters, and the activation of receptors constitute the means whereby nerve cells communicate and transmit information to one another and to non neuronal tissues[1].

2 Nerve Signal Transmission

Nerves transmit information by using electrical signals that travel down the cell membrane. These signals are called action potential. A nerve impulse is an all or nothing response : it is either on or off like a switch. A very weak stimulus will not trigger an impulse at all, but a stronger stimulus will trigger the same size of impulse, no matter how intense the stimulus. A stimulus is an event that causes a response. For example, holding a flame to your hand will trigger a pain signal, the heat is the stimulus[2].
As long as the cell membrane reaches a certain threshold (the threshold potential), an action potential will be triggered. The strength of the stimulus is transmitted by changing the frequency of the impulse, therefore, weak stimulus will trigger the impulse slowly, but strong stimuli will cause the impulse to trigger rapidly. Think of flipping a light switch on and off again slowly versus quickly: the slow on-off would signal a weak stimulus, but a rapid on-off-on-off-on-off would signal an intense stimulus[2].

3 Fitzhugh Nagumo Model

The analysis of the Hodgkin Huxley equations is extremely difficult because of the nonlinearities and large number of variable. Mathematical analysis would be helpful if it is performed on simpler equations whose solutions shared the qualitative properties of those of the Hodgkin Huxley equations. Analysis of such simpler systems may lead to the discovery of new phenomena, which may be searched for the original system and also experimental preparations. Such a simplified system of equations has its origin in the work of Fitzhugh and Nagumo and is known as the Fitzhugh Nagumo equations [3],[4]. In other word, The Fitzhugh-nagumo equations is a simplified form of the Hodgkin-Huxley model for electrical activity in a neuron. In this model a neuron can be stimulated with an input such as an electric current. The state of this excitation is described by variable $u_1$ which represent the voltage (excitation) in the neuron as a function of time. When a neuron is excited, physiological processes in the cell will cause the neuron to recover from the excitation. The variable $u_2$ in the model equation represents this recovery [3],[5]. The equations are given by:

$$\frac{\partial u_1}{\partial t} = \Delta u + (\alpha - u_1)(u_1 - 1)u_1 + (-u_2) + I$$  \quad \ldots (1)

$$\frac{\partial u_2}{\partial t} = \epsilon (\beta u_1 - \gamma u_2 - \delta)$$  \quad \ldots (2)

$\alpha$ is the excitation threshold and $\epsilon$ is the excitability. $\beta$, $\gamma$ and $\delta$ are parameters effecting the resting state and dynamics of the system [6].

4 Simulation

Comsol Multiphysics is one of the most important software in analysis and simulate the finite element method (FEM). This software has several characteristics that the most important of them is speed and accuracy that can be noted. Steps in the comsol multiphysics include Select the module and dimension, Design of model geometry, Determine the Subdomain Settings and boundary conditions, Select the materials and its characteristics, Mesh generation and Choose the type of analysis. Also in order to avoid numerical limitation and obtain more precise results, the values of the piezoelectric blade, axon and ionic environment are larger than actual values[3].

4.1 Select the module and dimension

The first step in simulation is suitable choice of the problem dimensions and the module which is needed. According to the model geometry, dimensions can be one, two or three dimensions. Also in a simulation, depending on the type of problem, several different modules may be used. Hence, in this simulation, in order to simulation of axon, we have used three dimensional structure and PDE (general form) module[7].
4.2 Design of model geometry

In this section, we create the geometric model of various components. In finite element method simulation, to reduce the time required for analysis, we use the appropriate geometric approximation to limit the size of the model. To simulate the axon, we have used from fitzhugh nagumo model. The axons geometry is a hollow cylinder with length of 125 m and radius of 5 cm [3],[7].

Fig. 1: Geometrical model for simulation

4.3 Determine the Subdomain Settings and boundary conditions

In PDE module, the axon subdomain described by two dependant variables, \( u_1 \) and \( u_2 \). The equation that solved by PDE mode is as following.

\[
e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \Gamma = F
\]  

...(3)

where \( e_a \) is mass coefficient, \( d_a \) is damping coefficient, \( \Gamma \) is numerical flux and \( F \) is source term. In order to create the main equations ie (1) and (2) equations, we need to following parameters [3],[7].

\[
e_a = 0 , \quad d_a = 1 , \quad F = 0
\]

The numerical flux \( \Gamma \) for equation (1) and (2) is set to:

\[
\Gamma = \Delta u + (\alpha - u_1)(u_1 - 1)u_1 + (-u_2) + 1
\]  

...(4)

\[
\Gamma = \varepsilon (\beta u_1 - \gamma u_2 - \delta)
\]  

...(5)

Boundary conditions for the axons as well as the following:

\[
u_1(t_0) = V_0 . (x + d > 0) . (z + d > 0)
\]  

...(6)

\[
u_2(t_0) = nu_0 . (x - d > 0) . (z + d > 0)
\]  

...(7)

Nonlinear differential equations or Fitzhugh-nagumo as axons describe the cell membrane behavior with respect to the input that we applied to them. All of the axon boundaries in the PDE mode are taken as Neumann boundary condition and the equation that used in boundary mode is as following [3],[7]:

\[-n. \Gamma = G
\]  

...(8)
Where \( f' \) is numerical flux and \( G \) is source term. \( f' \) from Equation (5) is obtained and \( G = 0 \). all boundaries of ionic environment are at ground potential \( (V = 0) \), also all boundaries of the axon are selected as electrical potential with the coupling variable \( u_2 = V_0 \) \([3],[7]\).

4.4 Select the materials and its characteristics

As mentioned in the previous sections, in order to simulate the axon, FitzHugh Nagumo model is applied.

4.5 Mesh Generation

The most important part of simulation in Comsol software is mesh generation. Generally the software Comsol, first divide the model into smaller parts and solve the problem for every components, and then extended to the whole issue. In the mesh generation, the size of different elements should be considered carefully as to reduce the size of elements, the increased number of elements result in increased accuracy and computation time. By increasing the accuracy of the elements, the computation time is reduced\([3]\).

![Fig. 2: The meshed model](image)

In this paper, according to the type of simulation and conditions, we have used the triangular elements in the model. The reason is that by using the triangular elements, solving the problem is raised quickly. In axons, mesh generation also have been tried to use smaller elements in sensitive areas such as axons and piezoelectric blade common border. 7145 is the number of elements used \([7]\).

4.6 Choose the type of analysis

To solve the problem, static analyzer is used. also, to consider the dynamics of the action potential, the time dependent solver and Direct (SPOOLES) system is used.

5 Post Processing

According to the above processes and applying them in simulation, you can select the initial and final points of axon by domain plot parameters from the post processing menu, then see the result of generation and propagation of action potentials waveform inside of axon that have been shown in fig 3. Then, after solving the problem, by changing the time of solution through the plot parameters from post processing menu, we can see propagation of the wave of action potential over the length the axon is as follow.
Fig. 3: Action potential at the starting (green) and the ending (blue) points of the axon.

Fig. 4: Propagation of the wave of action potential over the length of axon at second 1

Fig. 5: Propagation of the wave of action potential over the length of axon at second 30
Fig. 6: Propagation of the wave of action potential over the length of axon at second 50

Fig. 7: Propagation of the wave of action potential over the length of axon at second 80
Fig. 8: Propagation of the wave of action potential over the length of axon at second 110

Fig. 9: Propagation of the wave of action potential over the length of axon at second 140
6 Conclusion

In this paper, in order to show the propagation of the wave of action potential over the length of axon, through it, nerve signals are transmitted between different organs, we simulated the Fitzhugh Nagumo model. For this purpose, first, we presented description of action potential, signal transmitting, Fitzhugh Nagumo model and its equations. After familiarizing with some of concepts, We began to explain the simulation processes and at the same time by using the PDE module in Comsol multiphysics, start to simulate the axon. After finishing the simulation steps and obtaining the results, we saw that the wave of action potential form produced in the initial point of axon and by increasing the time of solution, this wave propagated over length of the axon and reach to the end of it. Thus, we have shown, the wave of action potential how to propagated inside of axon.

7 Appendix

Table 1. Constants for Fitzhugh Nagumo equations that used in the simulation [3],[7].

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.1</td>
<td>Excitation threshold [V]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.75</td>
<td>System Parameter</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1</td>
<td>System Parameter</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0</td>
<td>System Parameter</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.01</td>
<td>Excitability</td>
</tr>
<tr>
<td>$d$</td>
<td>1</td>
<td>Off-axis shift distance [m]</td>
</tr>
<tr>
<td>$V_0$</td>
<td>1</td>
<td>Electric potential</td>
</tr>
<tr>
<td>$nu_0$</td>
<td>0.025</td>
<td>Relaxtion value</td>
</tr>
</tbody>
</table>
8 References


